DEC 2009/JAN 2010

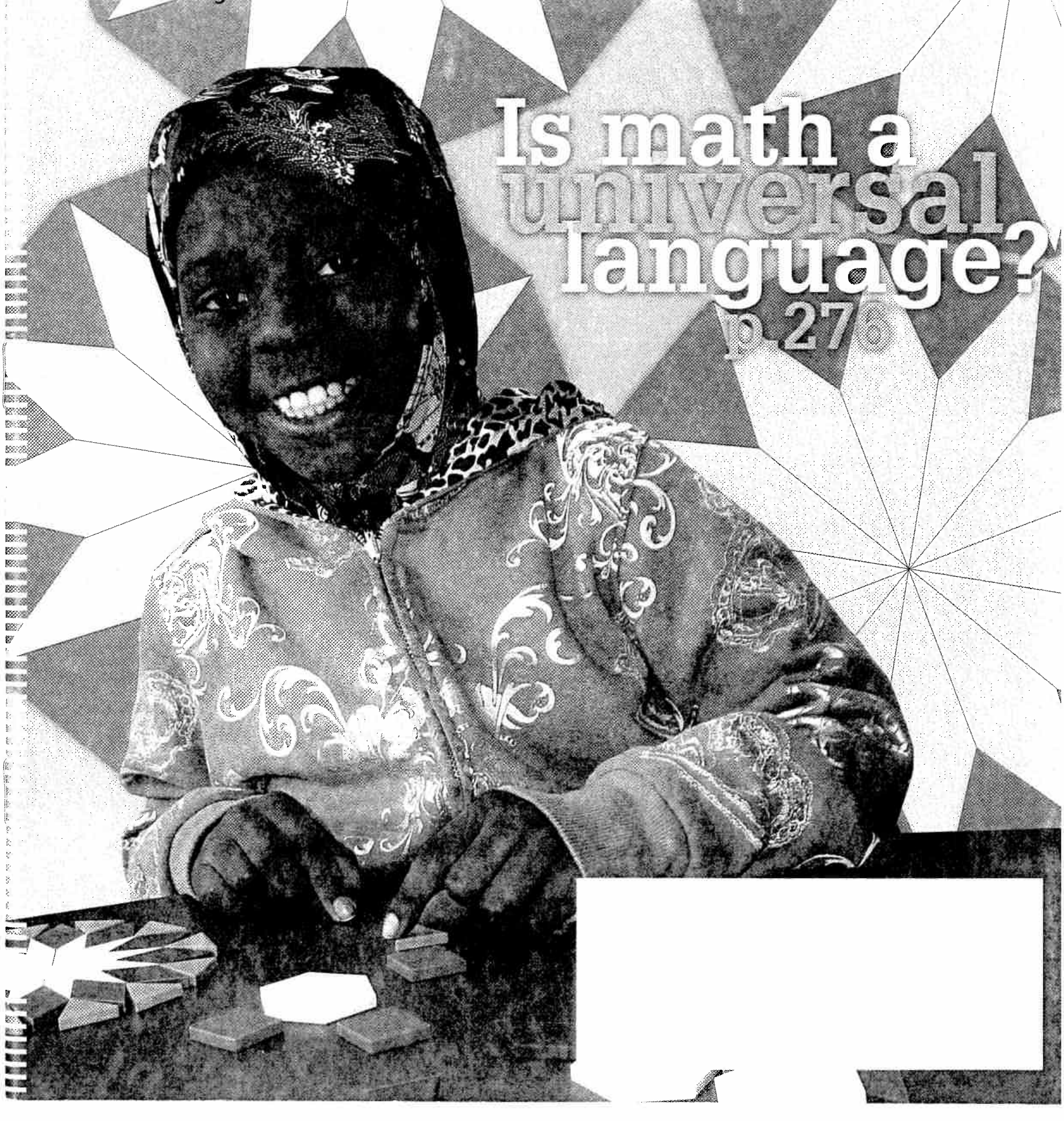
# teaching children mathematics

www.nctm.org

National Council of Teachers of Mathematics

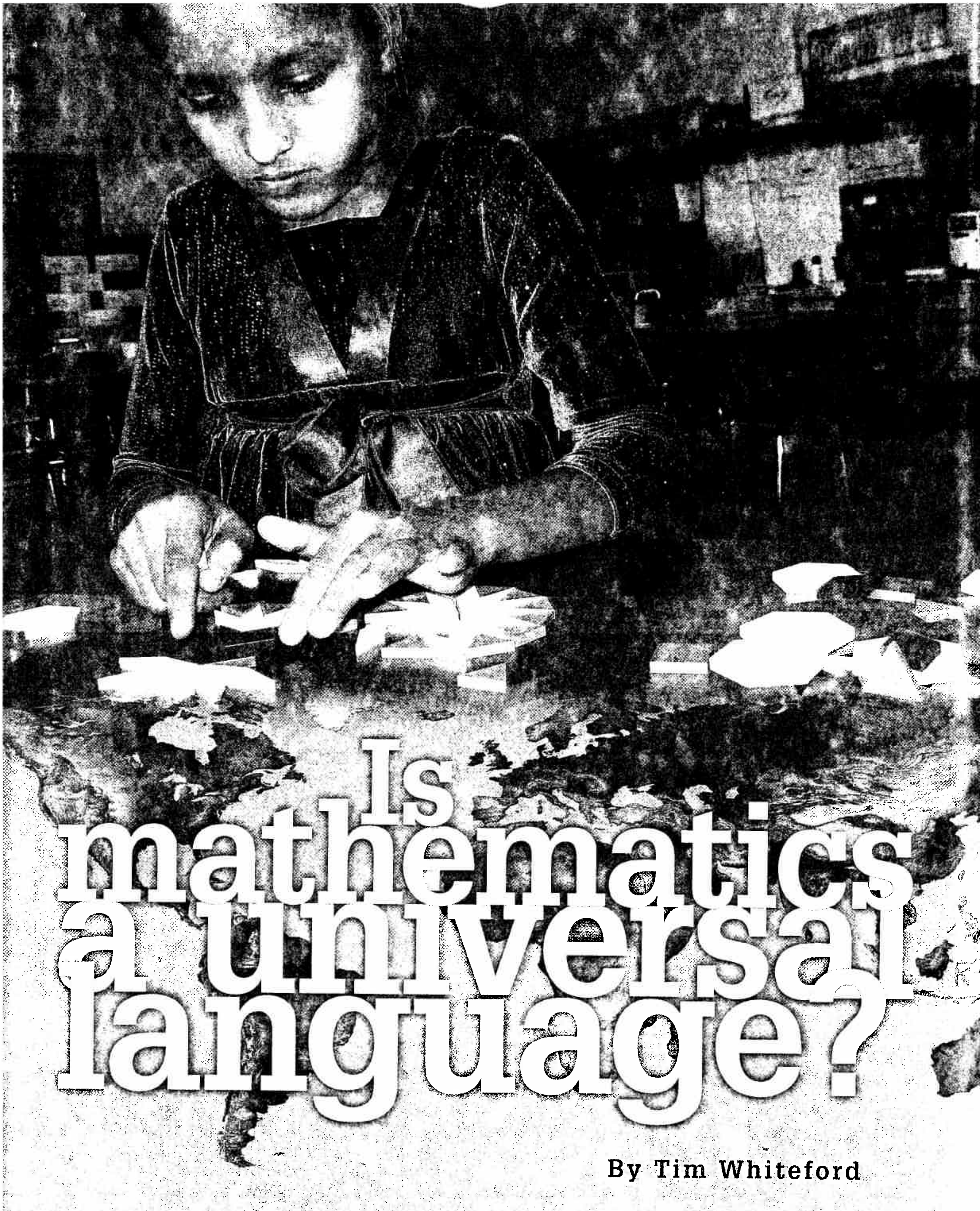
+ From calculating to calculus p.292

+ *The Polar Express* to early algebra p.300



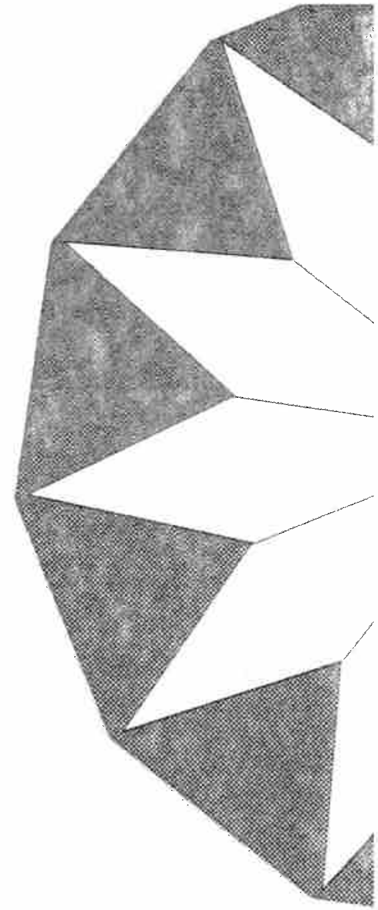
## Is math a universal language?

p.276



# Is mathematics a universal language?

By Tim Whiteford



## How mathematics is taught in other cultures has a significant impact on how students learn it when joining a U.S. classroom.

Melita is a sixth-grade student and a recent immigrant to the United States. She brings with her little knowledge of how to communicate in English but a wealth of mathematics that she has learned during the past five school years in her native Bosnia. Melita's mathematics differs from the mathematics that she will encounter in the coming years. She will recognize the numerals and the base-ten system, but she might find some of the procedures, such as the subtraction algorithm and the way that mathematics is taught, quite different from what she is accustomed to.

Melita will not be alone in this experience. Students from other countries, especially on the Asian and the African continents, may not find familiarity even in the numerals and counting system used in the United States. For students such as Melita, the challenges of learning mathematics can be as great as those encountered when learning to read, speak, and write in English. In the same way that we acknowledge a student's first language and culture, teachers must also acknowledge English Language Learners' (ELLs') "first mathematics," the math they bring with them to their new U.S. classroom. *This* is the mathematics that students will use to make sense of their new mathematical experiences. By acknowledging the differences between the two, we can more effectively help students make the transition to learning mathematics in American classrooms.

When students from other cultures enter the United States, the mathematics that they bring with them can be as different from the math that they are expected to use in the classroom as their native language is different from the English that they are expected to speak in the classroom. An ELL may have learned mathematics using a number base different from the base-ten system used in American culture (Zaslavsky 1993). The student may have learned to count using a different counting system (Durkin and Shire 1991) and may have learned mathematical procedures that are markedly different from those taught in American classrooms (Orey 2003). An ELL may also have learned mathematics in a classroom that stressed rote learning at the expense of developing students' understanding of what they were doing (M. Sedic-Lawton, personal communication January 26, 2009). These students most certainly have grown up experiencing the importance of numbers other than those that define the American culture: numbers such as 50 (states), 9 (innings in a baseball game), 12 (eggs in a dozen), 13 (original colonies), and so on. For instance, the number 3 is significant in Bosnian culture. In Ireland, 26 holds special meaning, signifying the number of counties in the northern part of the country.

As teachers, we must honor, validate, celebrate, acknowledge, and do our best to understand the mathematics—as well as the math education experiences—that ELLs bring into our classrooms. The more we do so, the easier it is for us to differentiate instruction for ELLs and optimize their learning.

Hiebert (1986) identifies two types of mathematical knowledge: procedural knowledge and conceptual knowledge. *Procedural* knowledge is arbitrarily constructed through convention and general agreement by the members of a particular culture. Comprising an understanding of symbols and procedures, this is the type of knowledge that we use to perform computations and to communicate mathematical ideas. The greatest variation between cultures can occur in the area of procedural mathematical knowledge.

*Conceptual* knowledge, on the other hand, is knowledge of ideas and concepts, as well as the relationships and connections that exist between them. Conceptual knowledge is universal and follows the laws of the natural world. For example, measurement division can be viewed as repeated subtraction (van de Walle 2007, p. 153): $12 \div 4$ is the same as $12 - 4 - 4 - 4$. As another example, when we measure angles, we use *degrees*, which are measures of rotation as opposed to measures of distance.

TIM WHITEFORD

Gay (2000) identifies culturally responsive teaching as a strategy for acknowledging students' cultural knowledge and prior experience. Using these two dimensions, in the context of mathematics, we can explore differences in ELLs' procedural knowledge and conceptual knowledge, in the number systems with which they might be familiar, in the nature of their cultural mathematics, and in ways that they learned mathematics in school.

## Procedural knowledge differences

In terms of Melita's procedural knowledge, for instance, she will most likely bring the equal-addition method of subtraction with her to her new American classroom. The method involves adding ten ones to the top number and one ten to the bottom number (see **fig. 1**). She would say, somewhat metaphorically, "We cannot take nine from two, and so we must borrow ten ones. We must then pay it back to the number in the tens place in the bottom number. We place the borrowed ones next to the two to make twelve and place the paid-back ten next to the five in the tens place to make six tens."

Melita's algorithmic procedure works because the difference between the two numbers remains the same: We add ten ones to the top number to make it eighty-two, and we add one ten to the bottom number, making it sixty-nine. This method was one of several used in the United States until the 1940s (Ross and Pratt-Cotter 1999), when the decomposition method, more familiarly called *regrouping*, was introduced as a more logical procedure.

Grammatically, the word *ten* is used as an adjective in the ones place (ten ones) and as a noun in the tens place (one ten). We still use—frequently and somewhat anachronistically—the *borrow* language instead of the more conceptually correct term *regroup* when we employ the decomposition method.

**FIGURE 1**

This algorithmic procedure works because the difference between 72 and 59 is the same as the difference between 82 and 69.

| | | |
|---|---|---|
| | 7 2 (+10) | Add 10 ones to the 2 ones to make 12. |
| (+10) | −5 9 | Add 1 ten to the 5 tens to make 6 tens. |
| | 1 3 | |

Another common difference in procedural knowledge is in the way that long-multiplication problems can be completed by multiplying by the tens first, as opposed to multiplying the number in the ones place (see **fig. 2**).

Today, in many American classrooms, students are taught a variety of computational methods, depending on the math program being used. Most standards-based programs advocate the use of invented algorithms in which students develop algorithmic procedures on the basis of their enhanced understanding of numeracy and the base-ten system.

The focus on the development of a student's mathematical understanding as opposed to the rote memorization of procedural knowledge is an aspect of education in the United States that ELLs and their parents may find difficult to embrace. I once observed a student teacher presenting a mathematics lesson in a culturally diverse fourth-grade classroom. The South Vietnamese community-based liaison worker in the classroom suddenly stood up and told the student teacher she was wasting everyone's time trying to get students to understand the material when all they needed to do was to memorize the procedures.

Be aware of the cultural differences that exist in the way that people deal with procedural knowledge. Sensitivity can help guide your instruction, especially when you teach algorithmic procedures. For students who have developed proficiency and automaticity with algorithmic procedures through rote instruction, developing standards of invented algorithms may not be worth the time it takes to bring about the required changes in procedural knowledge. Such time would be better spent helping students understand the procedure they are using or developing their problem-solving skills. Having grown up in the United Kingdom in the 1950s, I still, to this day, use the equal-addition method of subtraction when I complete my own subtraction algorithms.

In the field of ethnomathematics, resources are available, especially online, to help us understand how students from other cultures complete algorithms. The Algorithm Collection Project (Orey 2003), for example, is an unfinished attempt at gathering together in one place the variety of algorithmic procedures used around the world. An interesting addition to such a collection would be the inclusion of the words and phrases that students use as they complete their algorithms. Are they the same as those that U.S. students use?

Several years ago I worked with a teacher who had a boy from India in her class. The student had great difficulty learning how to divide. Neither his teacher nor I could understand his difficulty until I had a conversation with his mother. As she explained to me how she does division, she used the phrase, "Twelve into four goes three times."

"Twelve into four" made perfect sense when this mother demonstrated how she could share twelve pencils among four people so that each person receives three pencils. Such a linguistic construct would be more difficult to understand in a U.S. classroom, where we would typically say, "Four into twelve goes three times." Both phrases are mathematically correct, but both suffer to a certain degree from what Pimm identified as "teacher patter" (Pimm 1987): a short phrase in the United States that is designed to accompany, usually, a piece of procedural knowledge such as an algorithm. Another example of teacher patter is, "You cannot take six from four" when solving the problem eighty-four minus twenty-six. It is, of course, perfectly possible to subtract six from four to yield a result of negative two.

## Mathematical thinking differences

Although the conceptual knowledge of mathematics may not differ significantly between one culture and another, the way we access an ELL's conceptual knowledge can present significant difficulties. One challenge is to know exactly what the student is thinking mathematically in her first language. An effective way to do so is through an informal assessment designed to give the teacher insight into a student's precise mathematical understanding. Such assessments (e.g., Allsopp et al. 2008) can be constructed when teachers simply ask the student to complete a few activities in a targeted area and then engage the student in a conversation related to how she completed the activities. In cases where the student has limited English proficiency, graphic organizers, drawings, pictures, and manipulative materials can be used to help overcome the restrictions of language, especially if an interpreter or bilingual student is not available.

**FIGURE 2**

Long-multiplication problems can be completed by multiplying by the tens first instead of multiplying the number in the ones place.

| | |
|---|---|
| 12 | |
| 12 | × |
| 120 | First put a 0 in the ones place, and then multiply the top number (12) by the number in the tens place in the bottom number (1). |
| + 24 | Now multiply the top number (12) by the number in the ones place (2). |
| 144 | Add the two numbers. |

Several years ago, a concerned third-grade teacher contacted me about a student from the Democratic Republic of the Congo who had just joined her class. The student, a French speaker, was struggling in math class, a situation that the teacher assumed was a direct result of the student's limited English proficiency. I agreed to interview the student to assess the level of her mathematical knowledge and her understanding of rational counting. I would then prescribe some developmentally appropriate activities for the teacher to use with the student.

We duly met in the school's learning center. After five minutes it was clear that neither my French nor the student's English were good enough for us to have a meaningful conversation. I enlisted the help of the learning center coordinator who, being from Montreal, spoke excellent Canadian French. We then conducted the interview with me asking questions in English and the Learning Center coordinator translating into French. Within fifteen minutes, I concluded that it was not the student's limited English proficiency that was causing her difficulties in mathematics but her underdeveloped math skills. This third grader could not count or number name (list the number words) past eight in either English or French. Her one-to-one correspondence was also weak, and she could not subitize any number patterns.

In this case, I was fortunate to have the assistance of the French-speaking learning center coordinator. In other similar situations, I have been able to use pictures, drawings, and manipulatives to ascertain similar mathematical difficulties with students who had limited English proficiency.

If you choose to use an interpreter or bilingual student as illustrated in the previous example, then the interpreter must precisely communicate the meanings conveyed by the student, including the use of any metaphor or "teacher patter" (Pimm 1987) referred to earlier that the student may have picked up along the way.

Math education is full of metaphors, such as "reducing fractions" and "the face of a clock." In everyday usage, the term *reduce* means to make smaller. In the context of fractions, the term is used metaphorically, because nothing is actually getting smaller when we write the fraction in simplest terms except the numbers we use. One-half is *not* smaller than four-eighths. As van de Walle observes in his excellent math education text, "Notice how the phrase 'reducing fractions' was not used. This unfortunate terminology implies making a fraction smaller and is rarely used any more in textbooks" (2007, p. 312).

To really understand the nature of a student's mathematical thinking, we need to know all the words, phrases, and word meanings that a student uses in relation to a particular concept or procedure. We must understand the student's native mathematical linguistic register (Halliday 1975)—words, phrases, and associated meaning—if we are to match instruction precisely to the student's current performance level. Using drawings, pictures, graphic organizers, manipulative materials, and even short dramatic performances can give great insight into the meanings that students have constructed with words in a spoken language that teachers may not understand.

## Teaching tips

Knowing the areas that give English Language Learners the most trouble in mathematics allows teachers opportunities to modify instruction and to help students make connections with what they already know.

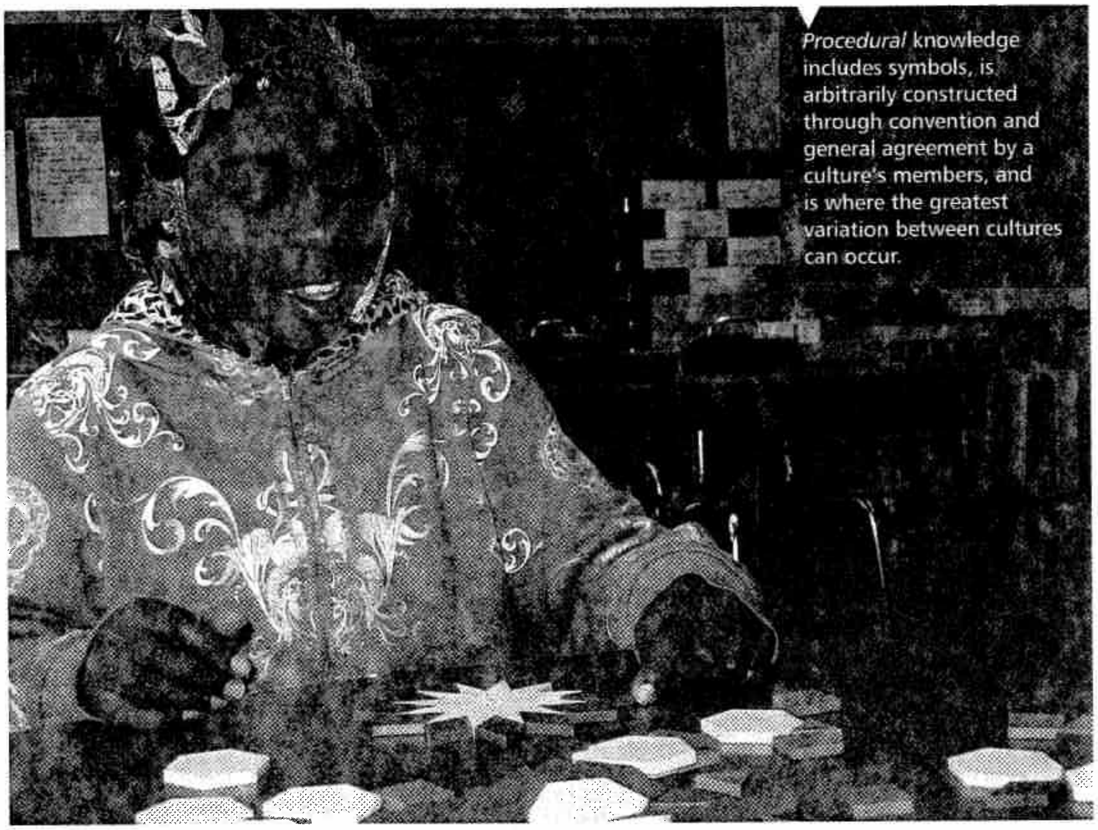
- **The ten-and-five** (15) structure present in many Asian languages relates to the English construction that uses the word *teen* to indicate ten.
- *Eleven, twelve* and teen number names in English differ in construction from single-digit number names.
- The *n* phoneme at the end of the teen number names distinguishes them from the decade names.
- **Referents**—such as *inches, feet,* and *yards* during a measurement class—must be explicit and clear; much of the meaning in the numbers we use is implicit in the context in which they are used.
- **Actual lengths** must be understood. They can be assessed better with activities
- **Culturally significant** numbers and referents help teachers to differentiate instruction, such as using currency from students' native cultures in word problems.
- **Approaches** to mathematics can differ by culture and can be discovered and investigated together by talking with students and doing online searches.
- **Web sites** provide activities to manipulate coins in other currencies and make change.

## Number systems and counting differences

Mathematics can differ between cultures as much as language itself, especially where different counting bases are used. Groups of indigenous people in the Amazon basin without counting systems use just three quantitative words for *one, two,* and *many.* On the African continent, some cultures use counting systems with base 5, 10, 12, 20, or in many cases, a composite of several different bases (Ascher 1991). Gesture counting, using very specific hand and finger referents, is also extremely important in many cultures (Zaslavsky 1993).

In other cultures where the base-ten system is used, the words used to identify certain quantities can be much easier to learn than the complexities of the English language caused by the words *eleven* and *twelve.* Many of the teen number names, such as *fifteen,* use modified single-digit number names followed by the suffix *teen* to cause further confusion for English Language Learners. The actual meanings of *eleven* and *twelve,* respectively, are *one remaining after ten* and *two remaining after ten.* Their roots are in the ancient *endleofan* and *twalef.* According to Online Etymology, Lithuanian is the only other language that uses this construction for teen numbers all the way to twenty (Harper 2001, www.etymonline.com/index.php?search=elevenandsearchmode=none).

The Maay Maay language of Somali Bantu students has, like many Asian languages, the simplicity of *tummung I kow* (ten and one) for eleven, *tummung i lammih* (ten and two) for twelve, and so on all the way to twenty (Cultural Orientation Resource Center, www.cal.org/co/bantu/sbgloss.html). Other cultures also avoid the incredibly confusing English similarities between the teen and decade names such as the homophonic *fifteen* and *fifty,* which



*Procedural* knowledge includes symbols, is arbitrarily constructed through convention and general agreement by a culture's members, and is where the greatest variation between cultures can occur.

TIM WHITEFORD

are made even more difficult by certain speakers' lack of experience pronouncing the *n* phoneme at the end of a word.

If we know something about the mathematics that ELLs bring with them into our classrooms, we can help students make connections with what they already know. More important, being aware of areas that cause the greatest difficulty, we can modify our instruction:

- **Highlight** and draw students' attention to the words *eleven, twelve*, and the teen number names that follow, as well as how they differ from the single-digit number names.
- **Stress the *n*** phoneme at the end of the teen number names so that students do not confuse them with the decade names.
- **Connect** the relationship between the ten-and-five (15) structure in many Asian languages and the English construction of the *teen* words that use the word *teen* to indicate ten.

## Cultural mathematics differences

One way of describing or defining a culture is to consider the referents that members of that culture use when measuring or quantifying. For instance, in North America, *pounds, ounces, pints*, and *gallons* are American, whereas *centimeters, liters*, and *grams* are Canadian. Culturally responsive teaching means recognizing the importance of including students' cultural references in all aspects of learning (Ladson-Billings 1994).

There are storekeepers in the United Kingdom who, some thirty years after compulsory decimalization, still use—somewhat illegally—imperial measures to sell their wares. The British pint of ale, for example, will probably never be superseded by the metric *liter*, although it is the customary measure for all other types of liquid, including petrol (gasoline) and milk.

The delay in universal acceptance of the metric system in the United States is probably due to the size of the country and the lack of collective political or social will to change. Everything we do in our lives is characterized in some way by the use of numbers, quantities, shapes, dimensions, and the language associated with such mathematical phenomena.

The use of *naked* numbers, those with no referent attached, occurs all the time in our daily lives but in no way detracts from communicating meaning if we are familiar with the context in which the numbers are used. *Two seventy-five, three-thirty, deep six, lower forty*, and *hitting three seventy-five* are all perfectly understood despite no referent for any of the numbers.

But phrases such as, "It used to cost four and eleven," and, "He was five for eighty-six at the close of play," are completely meaningless to anyone unfamiliar with the now-obsolete British currency of shillings and pennies or who has never experienced a cricket game. (*Four and eleven* is four shillings and eleven pennies; *five for eighty-six* refers to a cricket bowler's statistics: taking five wickets for eighty-six runs). So much of the meaning in the numbers we use is implicit in the context in which we use them.

This implied meaning of the referent impacts the way we teach ELLs in two ways. First, we must make explicit the presumed or inferred meaning associated with the numbers we use when students may be unfamiliar with the context.

- **Use referents** (e.g., *inches*, *feet*, and *yards*) to increase students' awareness during a measurement class.
- **Incorporate activities** to access whether students have a sense of actual lengths (e.g., How long is an inch?).

Although we now teach both imperial and metric forms of measurement in schools, metric measure usage is still an uncommon practice in our culture. Unlike most immigrants to the United States, native speakers do not use metric measures in our daily lives and therefore do not have the implicit sense of metric quantities. To get a sense of the difficulties an ELL might face, try to describe your distance to work in kilometers or describe how many liters it takes to fill your car's gas tank. How much is a kilo of potatoes or a meter of fabric? Given a conversion chart, we could easily answer these questions, but most Americans do not automatically think in terms of metric measures.

Second, we must honor the significant numbers and quantitative relationships in our students' native cultures. We can do so by modifying the content of problem-solving activities or allowing ELLs to use numbers that have specific personal meaning or significance.

- **Differentiate** instruction by ensuring that word problems contain numbers and referents of particular significance to ELLs, such as the currency used in their native cultures.
- **Investigate** the mathematics used in students' cultures by talking with students and doing online searches.
- **Search** out and use Web sites that provide activities for users to make change and manipulate coins in other currencies.

## Instructional differences

Although mathematics education reform has taken place in many countries around the world, we cannot assume that students have experienced classrooms, instructional strategies, and materials similar to those prevalent in U.S, schools. Children in many classrooms around the world still sit at desks in rows, silently listening while a teacher lectures. Similarly, the use of instructional materials such as the ubiquitous Unifix® cubes may appear foreign to students who come to the United States with no formal education or who have experienced classrooms where such instructional materials are not used. Many developing countries still do not have inclusive practices in their schools for students with special needs (Eleweke and Rodda 2000).

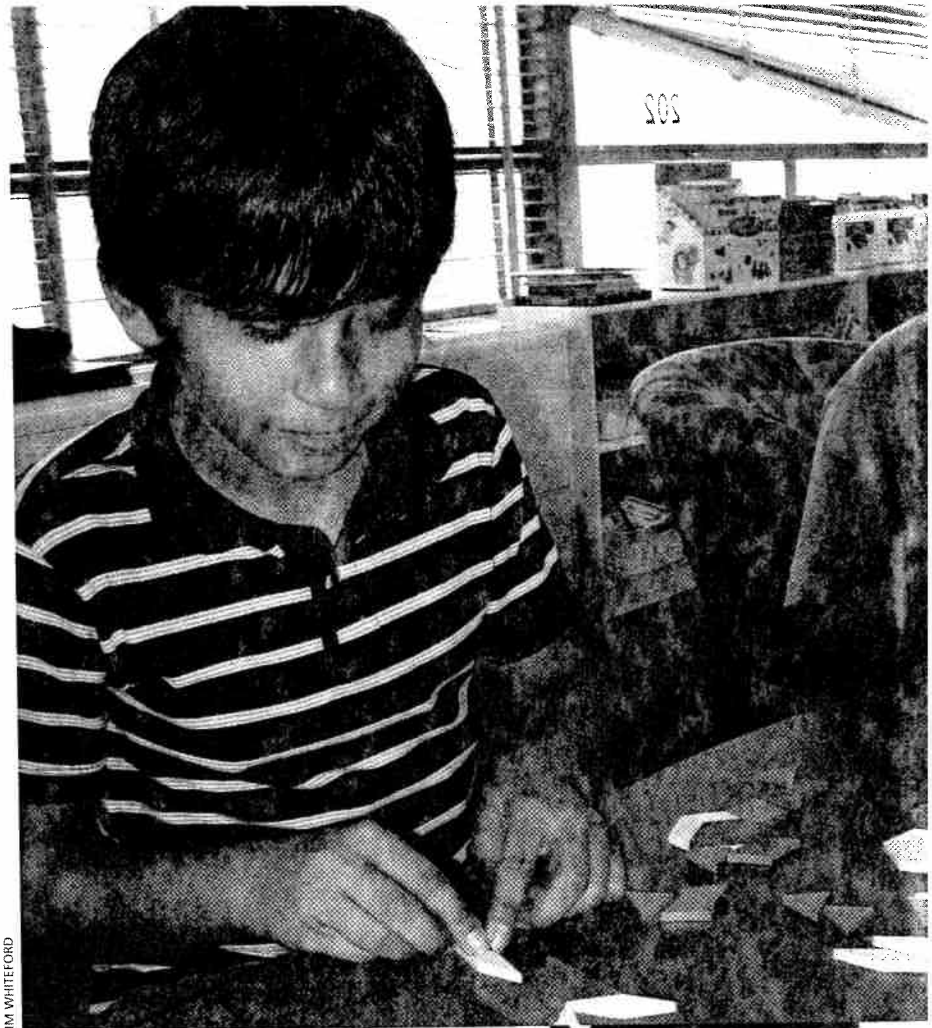
Knowledge of a student's mathematics education experiences, often gained from conversation with the student's parents, can help teachers be sensitive to particular instructional practices that a student may never have experienced and may not understand.

## Conclusion

Effectively teaching mathematics to speakers of other languages requires us to recognize, validate, honor, and support the math that these students have already learned before entering a U.S. classroom. To do so, we must become aware of procedures, types of math instruction, and students' current performance levels. We should be sensitive to cultural math differences that students may be experiencing. We need to make careful decisions as to whether students can continue to use procedures that differ from those taught in U.S. classrooms. We must take care not to assume that a student encountering difficulties in mathematics does so because of limited language proficiency.

Once we have considered these issues, we can then identify and implement ways of modifying, adapting, or differentiating classroom experiences. In addition to the sample ideas and strategies identified in this article, more extensive strategies exist, such as sheltered instruction (Echevarria, Vogt, and Short 2007).

Most of all, we ought to celebrate the diversity that English Language Learners bring to U.S. mathematics classrooms: other cultures' numbers, mathematical games, different number bases, and mathematical procedures. Recognizing cultural aspects and variations will help all of us better appreciate the multidimensional nature of mathematics.


TIM WHITEFORD

Much of the meaning in the numbers we use is implicit in the context in which we use them.

## REFERENCES


Allsopp, David H., Maggie M. Kyger, LouAnn Lovin, Helen Gerretson, Katie L. Carson, and Sharon Ray. "Mathematics Dynamic Assessment: Informal Assessment That Responds to the Needs of Struggling Learners." *Mathematics Teaching Exceptional Children* 40, no. 3 (2008): 6–16.

Ascher, Marcia. *Ethnomathematics: A Multicultural View of Mathematical Ideas*. New York: Chapman and Hall, 1991.

Cultural Orientation Resource Center. 2009. http://www.cal.org/co/bantu/sbgloss.html.

Durkin, Kevin, and Beatrice Shire, eds. *Language in Mathematical Education: Research and Practice*. Milton Keynes, UK: Open University Press, 1991.

Echevarria, Jane, MaryEllen Vogt, and Deborah J. Short. *Making Content Comprehensible for English Language Learners*. Boston: Pearson Education, 2007.

Eleweke, C. Jonah, and Michael Rodda. "Enhancing Inclusive Education in Developing Countries." Paper presented at the International Special Education Congress (ISEC), Manchester, England, 2000. http://www.isec2000.org.uk/abstracts/papers_e/eleweke_1.htm.

Gay, Geneva. *Culturally Responsive Teaching: Theory, Research, and Practice*. New York: Teachers College Press, 2000.

Halliday, M. A. K. *Learning How to Mean*. London: E. Arnold, 1975.

Harper, Douglas. Online Etymology Dictionary, November 2001. http://www.etymonline.com/index.php?search=elevenandsearchmode=none.

Hiebert, James. *Conceptual and Procedural Knowledge: The Case of Mathematics*. Mahwah, NJ: Lawrence Erlbaum Associates, 1986.

Ladson-Billings, G. "Who Will Teach Our Children? Preparing Teachers to Successfully Teach African American Learners." In *Teaching Diverse Learners: Formulating a Knowledge Base for Teaching Diverse Populations*, edited by Etta R. Hollins, Joyce E. King, and Warren C. Hayma, pp. 129–58. Albany: State University of New York Press, 1994.

Orey, Daniel Clark. "The Algorithm Collection Project (ACP): An Exploration of the Ethnomathematics of Basic Number Sense Acquisition across Cultures." Paper presented at the third conference of the European Society for Research in Mathematics Education, Sacramento, 2003. http://www.csus.edu/indiv/o/oreyd/papers/cerme.htm.

Pimm, David. *Speaking Mathematically*. London: Routledge, 1987.

Ross, Susan C., and Mary Pratt-Cotter. "Subtraction from a Historical Perspective," *School Science and Mathematics* 9, no. 7 (November 1999): 389–93.

Van de Walle, John. *Elementary and Middle School Mathematics: Teaching Developmentally*. 6th edition. Boston: Pearson, 2007.

Zaslavsky, Claudia. *Africa Counts*. Chicago, IL: Lawrence Hill Books, 1993.



**Tim Whiteford, twhiteford@smcvt.edu,** teaches graduate and undergraduate math and science education courses in the Education Department at Saint Michael's College in Colchester, Vermont. His interests include the role of language in teaching and learning mathematics at K–8 levels and the development of effective math education experiences for English Language Learners.